Stages in Division

Division – Stage 1

Children will engage in a wide variety of songs and rhymes, games and activities.
In practical activities and through discussion they will begin to solve problems involving halving and sharing.
Share the apples between two people.
'Half of the apples for you and half of the apples for me.'

Division – Stage 2

Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
Count in multiples of twos, fives and tens (to the 10th multiple)

Children will start with practical sharing using a variety of resources.
They will share objects into equal groups in a variety of situations.
They will begin to use the vocabulary associated with division in practical contexts.

'Share these eight apples equally between two children. How many apples will each child have?'

'Share these eight apples equally between two children. How many apples will each child have?'

'Share 20 crayons between 2 pots.'

'How many crayons are in each pot?'

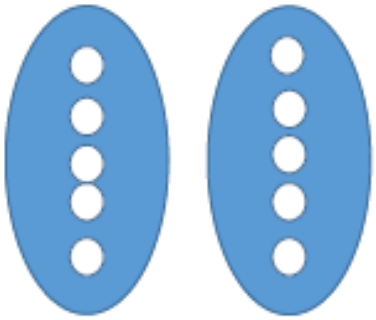
Children will move from **sharing** to **grouping** in a practical way

'Put 20 crayons into groups of 10. How many pots do we need?'

Use arrays to support early division

'How many faces altogether? How many groups of two?'

'Five groups of two.'



'How many groups of 5?'

'10 shared equally between 2 people.'

'Half of ten is five.'

Continue to solve problems in practical contexts throughout stage 2, and develop the language of early division, with appropriate resources.

Division – Stage 3

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
Calculate mathematical statements for division within the multiplication tables they know and write them using the division (÷) and equals (=) signs
Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts

NB Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

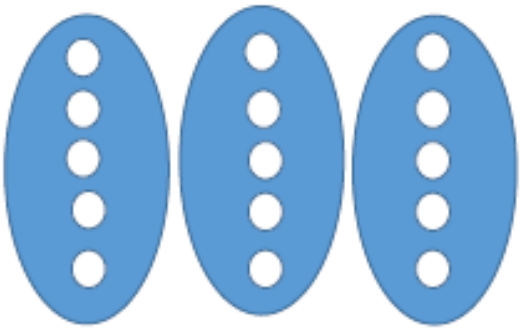
Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the ÷sign to record, using multiples that they know.

Sharing and grouping:

'30 crayons shared equally between 3 pots.' (Sharing)
'We have 30 crayons and put ten in each pot. How many pots do we need?' (Grouping).

30 ÷10 = 3
30 ÷ 3 = 10

'30 divided by 10 = 3'
'30 divided by 3 = 10'



'How many groups if 5?'

'15 shared equally between 3 people is...?'

'15 divided by 3 equals 5'

'15 divided by 5 equals 3'

15 ÷ 5 = 3
15 ÷ 3 = 5

Using arrays to support division

15 ÷ 5 = 3
15 ÷ 3 = 5

How many groups of 3?

How many groups of 5?

15 shared between 3 people is…?

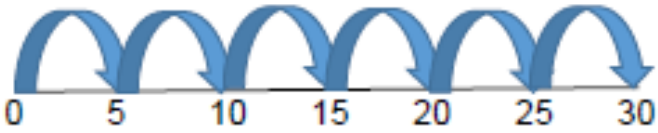
15 shared between 5 people is…?

15 divided by 5 = 3
15 divided by 3 = 5

When children are ready, use an empty number line to count forwards:

30 ÷ 5 = 6

'How many jumps of 5 make 30?'



0 5 10 15 20 25 30

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Division – Stage 4

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method

NB Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to use practical resources, pictures, diagrams, number lines, arrays and the ÷ sign to record, using multiples that they know, as appropriate (see stage 3 guidance).

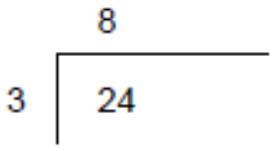
Using an empty number line to count forwards…
24 ÷ 3 = 8
'How many threes are in 24?'

'How many groups of three in 24?'

Introduce the formal layout using multiplication/division facts that the children know:

24 ÷ 3 = 8

This can also be recorded as…



'Twenty four divided by three equals eight.'
'How many threes are there in twenty four?'

NB If, at any time, children are making significant errors, return to the previous stage in calculation

Division – Stage 5

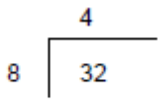
Recall multiplication and division facts for multiplication tables up to 12 × 12
Use place value, known and derived facts to divide mentally
Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programmes of study but implied in the non-statutory guidance)

NB Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

32 ÷ 8 = 4

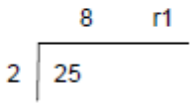
Continue using the **formal written layout** for division using multiplication tables that they know:



'How many eights are there in 32?'

Continue using the formal written layout, introducing remainders:

25 ÷ 3 = 8 r1



NB Remainders are not specifically referred to until year 5 in the National Curriculum. However, this may be an appropriate point to introduce them using familiar multiplication facts.

This could be modelled using an empty number line, if necessary:

Division using partitioning (two digits divided by one digit):

65 ÷ 5 = 13

65 = 50 + 15 Partition 65 into 50 and 15
50 ÷ 5 = 10
15 ÷ 5 = 3
10 + 3 = 13

NB Children will need to practise partitioning in a variety of ways.

98 ÷ 7 = 14

98 = 70 + 28 Partition 98 into 70 and 28

70 ÷ 7 = 10
28 ÷ 7 = 4
10 + 4 = 14

This could be modelled on an empty number line to further develop understanding.
NB Children will need to practise partitioning in a variety of ways.

This will lead into the formal written method of short division:

98 ÷ 7 = 14

$$\begin{array}{r} 14 \\ 7\overline{)98} \end{array}$$

Continue to practise the formal method of short division throughout stage 5.
If children are confident develop further, by dividing three-digit numbers by a one-digit number using the formal method of short division with whole number answers (no remainders).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Big Maths – don't forget to use the coin card method to help with number facts.

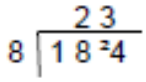
Division – Stage 6

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

NB Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to practise the formal written method of short division with whole number answers…

184 ÷ 8 = 23

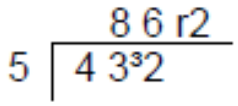


Use the language of place value to ensure understanding.

Make the link between the partitioning method (see stage 5 guidance)

…and with remainders:

432 ÷ 5 = 86 r2



The remainder can also be expressed as a fraction, 2/5 (the remainder divided by the divisor):

432 ÷ 5 = 86 2/5

Continue to practise, develop and extend the formal method of short division, with and without remainders. Interpret and express remainders according to the context.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Division – Stage 7

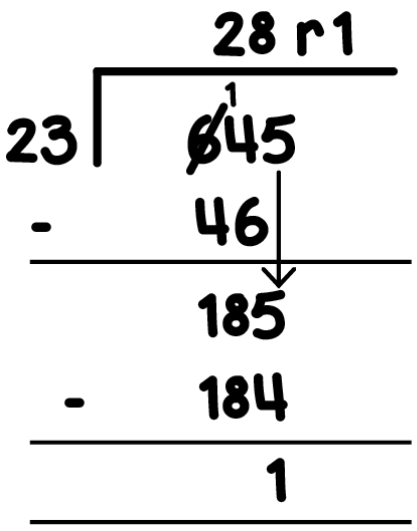
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

NB Ensure that children are confident with the methods outlined in the previous stage's guidance before moving on.

Continue to practise the formal method of short division, with and without remainders, using the language of place value to ensure understanding (see stage 6guidance).

3 digit by 2



This is the first time pupils divide by a 2d number. The new challenge this presents is that we can no longer use the 1d x 1d 'Learn Its' to solve division. However we do have other prior learning to lean on for success.

If pupils have been through the CLIC journey successfully then they will have gained the understanding necessary to underpin the column method from Step 5 of the 'Where's Mully?' Progress Drive from It's Nothing New.

Similarly, the pupil will also be able to quickly write out the first 10 multiples of the 2d number from the skills they developed in 'Coin Multiplication', and can also add multiples together to quickly find any multiple of the 2d number. This is extremely useful here since the pupil can then quickly answer the division challenge as they move along the columns (one could easily develop Coin Multiplication to the point where the child sees for themselves that they could continue to just find the highest multiple of the number without going past the target number in the question, and not need to keep doing it in 'bits' as they move along the columns – this is a natural extension of the thinking behind 'Where's Mully?').

x23	
1	23
2	46
3	69
4	92
5	115
6	138
7	161
8	184
9	207
10	230

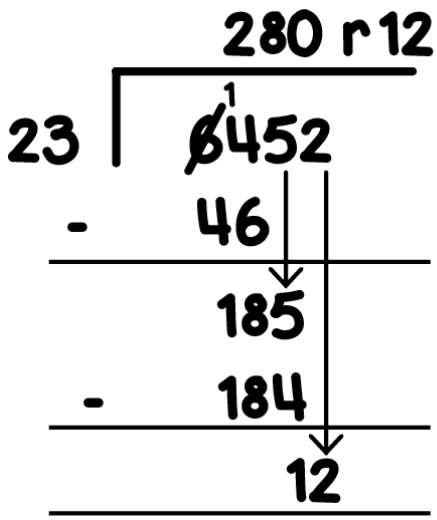
Just to be clear then, as soon as the learner sees the 2d number in the question they quickly write out their full coin card ready to use.

One further challenge is that sometimes the remainder for each individual column's division question is less easy to spot mentally compared to when we were dividing by 1d numbers. So, typically, the highest multiple of the 2d number is written under the digits in the question so that a simple column subtraction can be applied to find the remainder. At this point, instead of writing the remainder (i.e. the difference between the highest multiple of the 2d number and the number it is being divided into) as a mini-tens digit for the next column in the question, the digit from the next column in the question is brought down to place on the end of the remainder, and this is used to divide the 2d number into.

This process can then continue until we have found the whole number answer to the overall question.

An important progressive step is to then move on to provide questions where there is a remainder. This can just be presented as a remaining figure at this point, but it is useful to continue with the discussion of assessing the context of the remainder (when the question is set in a real life context).

4 digit by 2



This step has 2 mini-steps of progression:

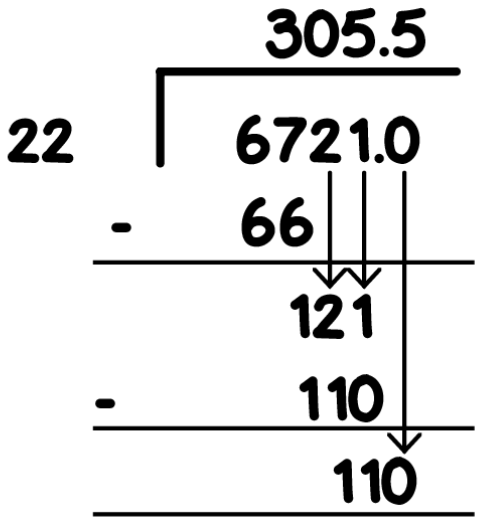
- Firstly to extend the skills of 3d divided by 2d into 4d divided by 2d (which requires no new skills just slightly greater numerical reward).
- Secondly to then take the remainder figure and represent it as a fraction. This is as easy as placing the remainder as a numerator with the 2d number from the division question as a denominator. In the example shown here the remainder would be:

$$\frac{12}{23}$$

- Choosing carefully constructed questions initially will allow for learners to access the understanding of why this shows the remainder as a fraction (for example, with 294 divided by 28 we find the answer is '10 remainder 14', looking at the 14 as a numerator over 28 as a denominator allows learners to see that it must be a half, i.e. we actually found that there are 10 and a half 28s in 294). It is also useful to choose questions where learners can then easily reduce the fraction into its simplest terms

x23	
1	23
2	46
3	69
4	92
5	115
6	138
7	161
8	184
9	207
10	230

I can solve division with decimal places in the answer



This last step sees the pupil now being asked to show the reminder as a decimal. In practice it is not as challenging as it may sound. After all, the children are used to the mechanics by now of whizzing along the columns, the only new skill is to take the final remainder and place it (as a mini-tens digit) in the tenths column of the question which is set up quickly by inserting a decimal point after the units column and then placing a zero in the tenths column.

It may be useful to retreat to division questions from earlier steps (e.g. dividing by 1d numbers) whilst this new skill is mastered and then returning to questions such as 4d divided by 2d after that.

Pupils ability should also be extended to questions that require 2 decimal places in the answer.

x22	
1	22
2	44
3	66
4	88
5	110
6	132
7	154
8	176
9	198
10	220

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Y6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an **efficient formal written method** accurately and with confidence.